Broadband anomalous reflection based on gradient low-Q meta-surface

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Gradient–index metamaterial is crucial in the spatial manipulation of electromagnetic wave. Here we present an efficient approach to extend the bandwidth of phase modulation by utilizing the broadband characteristic of low-quality (Q) meta-surface in the reflection mode. The dispersion of the meta-surface is engineered to compensate the phase difference induced by frequency change. Meanwhile, a thin gradient index cover layer is added on the top of meta-surface to extend the phase modulation range to cover the entire [0, 360°]. As a proof of concept, anomalous nearly perfect reflection with relative bandwidth near 40% is demonstrated in the microwave regime. © 2013 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution 3.0 Unported License. [http://dx.doi.org/10.1063/1.4809548]

As a novel material comprised of artificially engineered sub-wavelength structures, metamaterials (MMs) have many exotic electromagnetic properties including but not limited to negative refraction1 and strong anisotropy.2,3 In the last decade, researchers have explored the spectral and spatial properties of metamaterial for novel functionalities such as electromagnetic cloak4 and anomalous reflection/refraction.5 In the effective medium regime,6 the spatially controlled MMs can be treated as a gradient index material, which can be backdated to the establishment of Maxwell’s fish’s eye during 1860.7 Luneburg lens is another example to manifest the light manipulation power of gradient index.8 However, traditional gradient index cannot be found in natural occurring material due to its complexity in spatial distribution. Accompanied by the development of metamaterial and transformation optics in recent years,9 the performance of Luneburg lens and fish’s eye lens were improved to have either thinner thickness or better performance.10,11

More recently, two dimensional (2D) metamaterial surfaces (meta-surface) were adopted to achieve phase modulation in an ultrathin layer. For example, Yu et al. used a plasmonic nano-antenna array consisting of V-shaped gold to introduce phase discontinuity at the interface.5 Both anomalous reflection/refraction and optical vortex are demonstrated, followed by various designs using different approaches.12–16 In a related article, Zhou et al. proposed that an ultrathin gradient “magnetic resonance” can be utilized to convert propagating waves to surface waves in reflection mode.17 Since transmission is prohibited by the metallic ground plane, the energy efficiency can be much higher than the V-shaped structures.

Apart from the thickness requirement, another important design goal of gradient MMs is the broadband functionality, which means that the anomalous reflection, refraction and focus etc. should be maintained in a wide frequency band. Since the bandwidth of metamaterial is often limited
FIG. 1. (a) Schematic of the gradient meta-surface. (b) Unit cell of the multilayer structure. The top layer is added to introduce an additional phase shift.

by its resonant characteristic, techniques such as gradient multilayer and dispersion engineering are typically utilized to extend the working bandwidth.\textsuperscript{18–20} For anomalous reflection/refraction, broadband performance has been demonstrated to be possible through cross-polarized scattering from V-shaped array.\textsuperscript{21} Nevertheless, the scattered light possesses different polarization with the incident one and the efficiency is low (upper limit is estimated to $\sim 10\%$) because the V-shaped structure supports both anomalous and normal reflected/refracted beams.\textsuperscript{22} As parts of the efforts to overcome this defect, Zhou and Bozhevolnyi \textit{et al.} demonstrated that high efficient broadband anomalous reflection can be achieved by utilizing the magnetic resonance between metallic patch and ground plane.\textsuperscript{14, 16} It is believed that resonance with low quality factor (Q factor) is useful to extend the working bandwidth. Unfortunately, the phase change can hardly cover the entire $[0, 360^\circ]$ thus a quite large Q factor is used in these works.\textsuperscript{14, 16}

In this paper, we explore the concept of extremely low-Q meta-surface to achieve broadband linearly phase shift. Theoretic analysis demonstrates that broadband phase modulation is possible when the dispersion of meta-surface is properly designed within a certain range of phase shift. In order to extend the phase shift range to cover the entire $[0, 360^\circ]$, an additional gradient-index thin slab is added on the top of the low-Q meta-surface. As a proof-of-concept study, the approach is used to demonstrate broadband anomalous reflection in the microwave regime. Numerical calculations based on finite element method (FEM) and finite difference time domain (FDTD) demonstrate the working principle unambiguously.

As illustrated in Fig. 1, the gradient meta-surface proposed here has 10 unit cells along $x$ direction in each period (two periods are illustrated here). Each unit cell is comprised of a metallic ground plane, a dielectric spacer with permittivity of 2.5 and thickness of $d$, a meta-surface layer, and a cover layer (optional) with thickness of $t$ and tunable permittivity. The metallic ground plane
ensures that all incident electromagnetic wave would be reflected. The unit cell of meta-surface is composed of two vertically placed metallic wires connected by two horizontal stripes. It is chosen because its frequency dispersion can be easily tuned by changing the effective capacitance and inductance.\textsuperscript{17,20} The incident electric field is polarized along y direction, thus the meta-surface can be treated as an impedance sheet with $Z_\gamma(\omega)$. The reflection phase $\Phi_y$ can be controlled by changing $Z_\gamma$ and (or) the permittivity of the top layer. In principle, the gradient of $\Phi_y$ should be constant, that is, $\partial \Phi_y / \partial x = \xi$ for anomalous reflection. According to the generalized Snell’s law,\textsuperscript{3} the reflected beam will possess a parallel wave-vector $k_z = \xi$ for normal incident plane wave. The reflection angle can then be calculated as $\sin^{-1}(k_z/k)$.

It is well known that the gap width of the meta-surface can be utilized to tune the surface impedance and reflection phase.\textsuperscript{14,15,17} However, the physical mechanisms of phase change are different for different thicknesses of dielectric spacer. When $d$ is in deep-sub-wavelength scale, the magnetic coupling between the meta-surface and ground plane will dominate. In this case, $Z_\gamma$ can be characterized by a resonant LC parallel circuit.\textsuperscript{23} When $d$ is comparable with one quarter of the wavelength, the magnetic coupling disappears (the Q factor becomes extremely small) and multilayer interferences play the most important role in the phase change.\textsuperscript{19}

We first investigate the influence of gap width for small spacer thickness ($d = 2$ mm) when the cover dielectric layer is not taken into consideration. Each unit cell is modeled using periodic boundary condition using finite element method (FEM). The lengths of the unit cells along $x$ and $y$ directions are $p_x = 5.2$ mm and $p_y = 7.4$ mm. The metallic parts are chosen as copper with conductivity of $5 \times 10^7$ S/m and thickness of 0.017 mm. The width of the metallic parts are kept as $w = 0.2$ mm and $l = 4$ mm. As shown in Fig. 2(a), the phase difference for $g = 0.2$ mm and 6 mm is close to 360° in the frequency range between frequencies $f = 7$ GHz and $f = 15$ GHz. Nevertheless, the phase gradient $\partial \Phi_y / \partial x$ is not a constant in these frequencies thus the working bandwidth is essentially limited. As an example, one can see that the phase shift very fast (slow) at smaller gaps ($g = 0.2$-1.5 mm) at $f = 7 (15)$ GHz. The rapid frequency change of phase can be understood by a simple magnetic resonator model. Taking 10 GHz as an example, the meta-surface is highly reflecting and can be considered as a perfect electric conductor (PEC) when $g = 0.2$ mm, thus the reflection phase is -180° (the plane above the meta-surface is chosen as reference). When $g = 2.8$ mm, the magnetic resonance takes place at 10 GHz, and the reflection phase is just 0°. As plotted in Fig. 2(c) and 2(d), circulating electric fields are formed due to the magnetic coupling. Since the Q-factor of magnetic resonance is very high, the phase changes from 0 to 360° within a narrow frequency range. When $g = 6$ mm, the meta-surface becomes transparent and the reflection phase turns to be $180^\circ - \Phi_f$, where $\Phi_f = 2\pi kd$ is the phase shift due to the propagation inside the dielectric spacer. Obviously, the phase difference between $g = 0.2$ mm and $g = 6$ mm becomes closer to 360° for thinner dielectric spacer.

In order to achieve linear phase shift in a wider frequency range, the Q-factor of magnetic resonance should be suppressed dramatically.\textsuperscript{14} One direct approach is to increase the thickness of the dielectric spacer thus the magnetic coupling between meta-surface and ground plane becomes extremely weak. In the numerical simulation, the thickness is set as 6 mm which is corresponding to one quarter of the wavelength inside the dielectric spacer at 8 GHz. The reflection phases for different gap widths are illustrated in Fig. 2(b). As we expect, the phase change become near linear in the whole frequency range considered here (8–12 GHz). As shown in Fig. 2(e) and 2(f), the magnetic coupling between the meta-surface and ground plane is neglectable.

Unfortunately, the overall phase shift between $g = 0.2$ mm and $g = 6$ mm for $d = 6$ mm becomes only about 180° due to the increase of phase delay in the dielectric spacer. This limitation in phase modulation can be understood by considering the transfer matrix of the multilayer.\textsuperscript{20,23} According to previous supposition, the phase change in $x$ and $y$ directions can be written as:

$$\Phi_x \approx \pi + 2\pi kd$$
$$\Phi_y = \arg \left( \frac{1-n-Z_\gamma/Z_0+(-1-n+Z_\gamma/Z_0)\exp(2\pi kd)}{1+n+Z_\gamma/Z_0+(-1+n-Z_\gamma/Z_0)\exp(2\pi kd)} \right)$$

(1)

where $k$ is the wave vector in free space, $d$ is the thickness of dielectric spacer, $n = 1.58$ is the refractive index of dielectric spacer. In principle, a constant difference between adjacent phase shift
FIG. 2. Reflection phases for different values of gap width g for thickness of (a) $d = 2$ mm and (b) $d = 6$ mm. The other geometrical parameters are chosen as $l = 4$ mm and $w = 0.2$ mm. (c) y-component and (d) z-component of electric fields for $d = 2$ mm, $g = 2.8$ mm and $f = 10$ GHz. (e) y-component and (f) z-component of electric fields for $d = 6$ mm, $g = 2.8$ mm and $f = 10$ GHz.

$(\Phi_{y1} - \Phi_{y2})$ is required for broadband anomalous reflection. As $\Phi_y$ keeps unchanged along x direction, one can use it as a reference. Under this consideration, the optimal impedance for $\Phi_y - \Phi_x = \Delta$ can be written in an explicit form as:

$$Z_y = \frac{A - AB + 1 - B}{(1 - n)(1 - AB) - (1 + n)(A + B)}$$

where $A = \exp(i(\Delta + 2nkd + \pi))$ and $B = \exp(2nkd)$. Since no material loss is considered, $Z_y$ is imaginary and can be written as $Z_y = -iX$, where $X$ is the sheet reactance.

Using Eq. (2), the normalized optimal sheet reactance $X/Z_0$ for $\Delta = 0.5\pi$, $\pi$ and $1.5\pi$ are illustrated in Fig. 3, which changes from mainly capacitive to primarily inductive in frequencies between 6 and 12 GHz. The frequency dispersion should be used as an optimization goal in the design of meta-surface, similar with the case of broadband absorber and polarizer.\textsuperscript{19,20} In order to validate the above results, the effective sheet reactance of the meta-surface ($g = 0.2$ mm, 1.5 mm and 3 mm) are retrieved from the reflection and transmission spectra (the ground plane and substrate are not considered here). In the three cases, the retrieved reactance can be written in a form of series connected inductance and capacitance: $X(\omega) = \omega L - 1/\omega C$. The corresponding values can be directly fitted as $L_1 = 3.7$ nH, $C_1 = 0.145$ pF, $L_2 = 3.6$ nH, $C_2 = 0.06$ pF, $L_3 = 3.5$ nH, and $C_3 = 0.032$ pF.

A considerable effect shown in Fig. 3 is that the reflection phase changes almost acromatically as the change of effective sheet reactance. That is, the phase difference in a wide frequency band is a constant. For example, there is $\Delta \approx \pi$ for $g = 0.2$ mm within 4–12 GHz. For $g = 2.1$ mm (not shown...
FIG. 3. Normalized optimal sheet reactance for designated phase shifts ($\Delta = 0.5\pi$, $\pi$ and $1.5\pi$). The effective reactance for $g = 0.2$ mm, 1.5 mm and 3 mm are given for comparison.

here), there is $\Delta \approx 0.5\pi$ in frequencies between 8–13 GHz. This implies that the phase difference between $g = 0.2$ mm and $g = 2.1$ mm would be $0.5\pi$ within 8–12 GHz (see Fig. 2(b)). In fact, by tuning other geometric parameters the optimal sheet impedances can be approached more precisely.

Although the phase difference between adjacent unit cells can be designed to have linear gradient in a wide frequency range, the phase modulation range cannot cover the whole $[0, 360^\circ]$ because the change of sheet impedance is limited by the obtainable surface structure. In order to extend the phase modulation range, a thin dielectric layer ($t = 1$ mm) is added on the top of the meta-surface. By changing the refractive index $n_1$, an additional phase shift of about $\Delta_1 = 2n_1kd$ is introduced. Although this phase shift is essentially frequency dependent, the frequency dependence could be cancelled by properly designing the sheet impedance of meta-surface. The tuning of $n_1$ can be achieved by drilling sub-wavelength hole in a high refractive index slab. The effective permittivity of the holes array can be calculated using Maxwell-garnet theory:24

$$\varepsilon_{eff} = \frac{\varepsilon_{air}(1 + 2f) + 2\varepsilon_h(1 - f)}{\varepsilon_{air}(1 - f) + \varepsilon_h(2 + f)}$$

where $f = \pi r^2/a^2$ is the filling ratio of air, $r$ is the radius of hole, $a$ is the period of hole array.

Subsequently, one could combine the meta-surface with the gradient cover layer to realize broadband phase modulation in the range of $[0, 360^\circ]$. The parameters for 10 unit cells in one period along $x$ direction are optimized at 10 GHz and illustrated in Table I. When the permittivity is 1, the top cover layer can be omitted. The corresponding reflection phases $\Phi_i$ for different frequencies of the ten unit cells are illustrated in Fig. 4. Although there are some fluctuations compared with the ideal phases within 8–11 GHz, the phase differences between Cell-A and Cell-J are close to $324^\circ$, which are sufficient for most broadband functionality. Furthermore, we believe that more elaborate optimization can be used to achieve better results.

In order to verify the performance of the whole structure, finite difference time domain (FDTD) method is used to calculate the electric field distribution for TE polarization at normal incidence. As shown in Fig. 1(a), there are two periods (20 unit cells) in $x$ direction and 10 unit cells along $y$ direction. Perfect matched layer (PML) boundary conditions are used in both $x$, $y$, and $z$ directions. The total mesh used is approximately 73,000,000, which is sufficient for calculation precision. Fig. 5 depicts the electric field distributions for 8 GHz, 8.5 GHz, 9 GHz, 9.5 GHz, 10 GHz, and 10.5 GHz. For clarity of discussion, only the scattering region (not directly illuminated by the plane wave) is illustrated. Obviously, the reflection wave fronts are flat except for some edge diffraction for 8–10.5 GHz. The corresponding reflection angles are evaluated as: 46°, 43°, 40°, 37°, 35°, and 33°, agreeing well with the theoretically calculated results: 46.2°, 42.7°, 39.9°, 37.4°, 35.2°, and 33.3°.
TABLE I. Optimized parameters for the 10 unit cells.

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<th>Cell number</th>
<th>$\varepsilon_{\text{eff}}$</th>
<th>$g$ (mm)</th>
<th>$l$ (mm)</th>
<th>$w$ (mm)</th>
<th>$\Phi_1$ at 10 GHz (°)</th>
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FIG. 4. Reflection phases for different values of gap width $g$ when a dielectric cover layer is added on top of the meta-surface. The parameters are shown in Table I.

FIG. 5. (a) Schematic of the numerical calculation model. The dashed region represents the scattering region considered here. (b)-(g) show the Electric filed $E_y$ distribution in the scattering region at normal incidence for 8 GHz, 8.5 GHz, 9 GHz, 9.5 GHz, 10 GHz, and 10.5 GHz. The dashed black arrows indicate the propagation direction evaluated by $\sin^{-1}(k_A/k)$. 
In summary, we proposed an approach to realize broadband phase modulation in the range of $[0, 360^\circ]$. The dispersion of the meta-surface is engineered to compensate the phase difference induced by frequency change. In order to extend the phase modulation range limited by the obtainable sheet impedance, an additional thin gradient-index cover layer is utilized. As a proof of concept, broadband anomalous reflection of electromagnetic wave is demonstrated in the range of 8-11 GHz. It should be also commented that the broadband phase modulation approach proposed here can also be utilized in the design of broadband planar lens, electromagnetic vortex etc. Furthermore, although the device in this paper is designed for microwave frequency, it can be easily extended to terahertz and infrared frequencies.\textsuperscript{20} In the visible range, the plasmonic loss may deteriorate the performance.

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